Student Number/Name: ....

HSC Mathematics

Task 3 – 2014

Time Allowed - 1 hour + 5 minutes reading time

Multiple Choice	/4
Q5 Logarithms	/14
Q6 Trigonometry	/11
Q7 Trigonometry	/11
Total	/40

General Instructions:

- Write using black or blue pen
  Black pen is preferred
- Board-approved calculators may be used
- A table of standard integrals is provided
- In Questions 5 – 7, show relevant mathematical reasoning and/or calculations

1) What is the derivative of $\log_2 x$?

(A) $\frac{1}{x}$

(B) $\frac{1}{2x}$

(C) $\ln 2x$

(D) $\frac{1}{x \ln 2}$

2) What is $\int \frac{3x^2}{x^3+1} dx$?

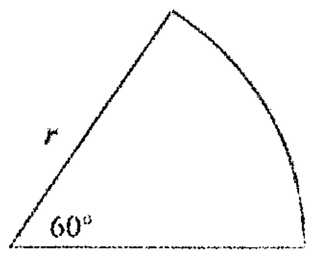
(A) $3\ln(x^3+1)+C$

(B) $\frac{1}{3}\ln(x^3+1)+C$

(C) $\ln(x^3+1)+C$

(D) $\ln 3x^2 + C$

3) The sector below has an area of 5π square units



What is the value of r?

(A) $\sqrt{60}$

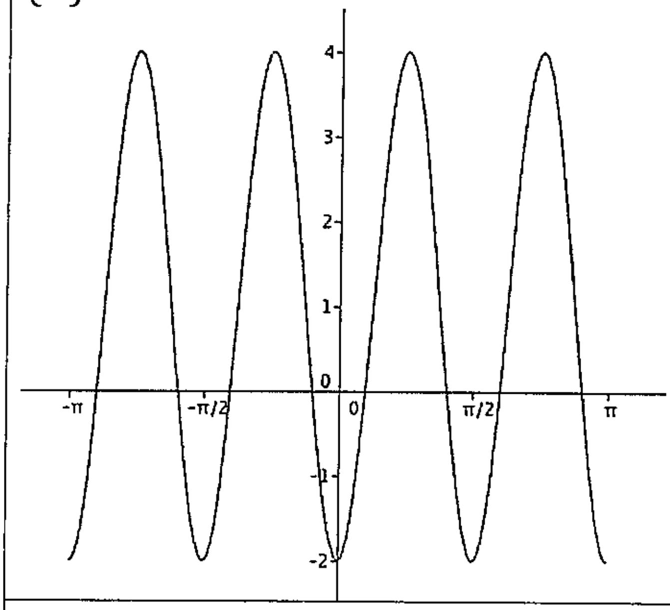
(B) $\sqrt{60\pi}$

(C) $\sqrt{\frac{\pi}{6}}$

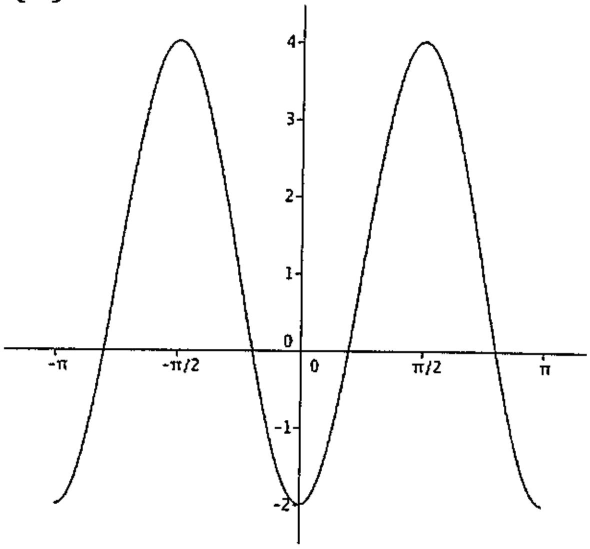
(D) $\sqrt{30}$

4) Which of the following is $y = 1 - 3\cos(4x)$ on the domain $-\pi \leq x \leq \pi$

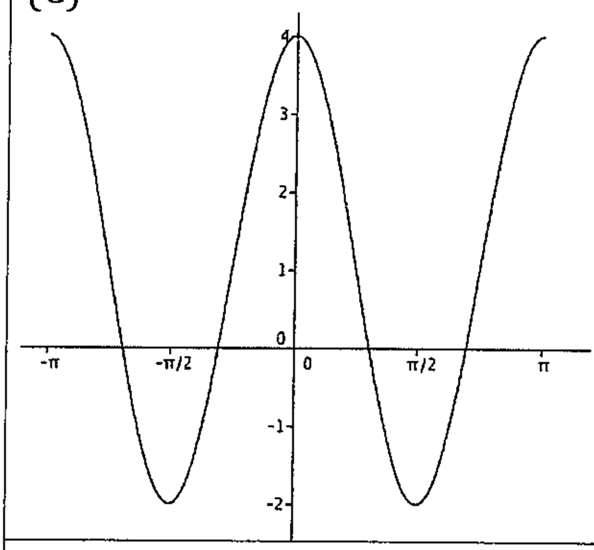
(A)



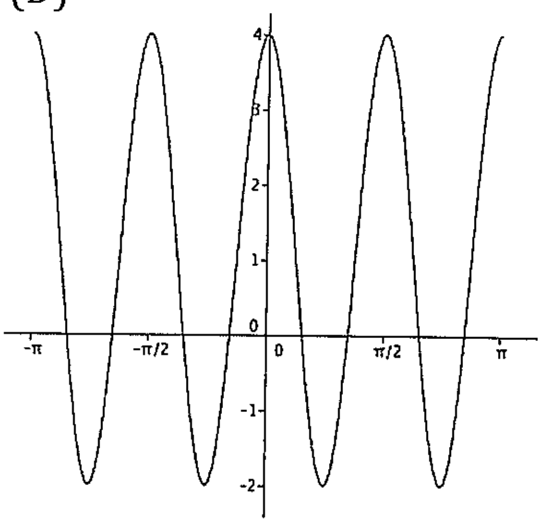
(B)



(C)



(D)



Qn 5) **Start a New Page**

a) Find $\frac{d}{dx}(\ln(x^2+3))$ 1

b) Differentiate $y = x\ln x$ 1

c) Evaluate $\int_{\sqrt{5}}^{3} \frac{x}{x^2-4}\,dx$ 2

d) It is known that the area bound by the function $f(x) = \frac{1}{x+1}$, the x −axis, $x = 0$ and $x = a$ is 3 units squared. What is the value of a? 2

e) Consider the function $f(x) = \frac{x}{\ln x}$

i. Show that $f'(x) = \frac{\ln(x)-1}{(\ln x)^2}$ 1

ii. Find the coordinates of any stationary point and determine its nature 2

iii. Show that the function has an asymptote at $x = 1$ 1

iv. Graph the function on the domain $1 < x \leq 5$ 2

f) Find $f'(x)$ for $f(x) = \ln\left(\frac{x+2}{x}\right)$ 2

Qn 6) **Start a New Page**

a) Convert $\frac{\pi}{5}$ radians to degrees 1

b) Find $\frac{d}{dx}(2\sin(3x-1))$ 1

c) Differentiate $\cos(e^x)$ 1

d) Find the gradient function of $f(x) = \frac{\sin x}{x}$ 1

e) Find the equation of the tangent to $y = \sin^2 x$ at $\left(\frac{\pi}{3}, \frac{3}{4}\right)$ 2

f) Find $\int \cos 3x \, dx$ 1

g) Evaluate $\int_0^{\pi} \sin\frac{x}{2} \, dx$ 2

h) Find the volume of the solid of revolution formed by rotating $y = \sec x$ around the x −axis from $x = 0$ to $x = \frac{\pi}{3}$ 2

Qn 7) **Start a New Page**

a) The **primitive function** of $f(x)$ is $\sin(2x)$. Given $f\left(\frac{\pi}{2}\right) = 2$ find the equation of $f(x)$? 2

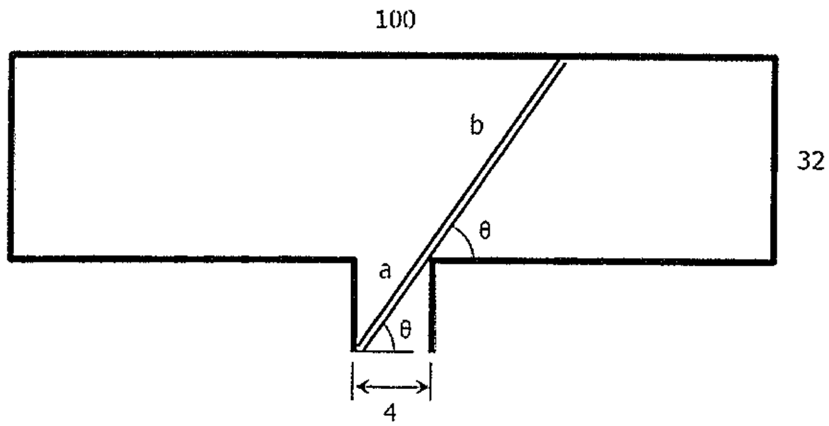
b) For the function $f(x) = 2\sin x$

i. Show that $f(x)$ is an odd function 1

ii. Hence, or otherwise evaluate 1

$$\int_{-\pi}^{\pi} 2\sin x \, dx$$

c) A corridor 4 m wide opens into a room 100 m long and 32 m wide, at the middle of the long side as illustrated. A pole of length $a + b$ m is to be carried in at an angel of θ to the doorway.



i. Show that the length of the pole is given by $L = 4\sec\theta + 32cosec\theta$ 1

ii. Show that $\frac{dL}{d\theta} = 4\tan\theta\sec\theta - 32\cot\theta\, cosec\theta$ 3

d) The tide can be modelled by the equation $h = A\cos Bt + C$ where h is the water depth and t is the number of hours from a high tide. If high tide occurs at 4am with a depth of 6 metres and low tide is at 10am with a depth of 2 metres. Find the values of A, B and C 3

HSC Mathematics

Student Name/Number:__________________________

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample:	2 + 4 =	(A) 2	(B) 6	(C) 8	(D) 9
		A ◯	B ●	C ◯	D ◯

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ⊗ C ◯ D ◯

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word correct and drawing an arrow as follows.

A ⊗ B ⊗ (correct ↓) C ◯ D ◯

1. A ◯ B ◯ C ◯ D ◯
2. A ◯ B ◯ C ◯ D ◯
3. A ◯ B ◯ C ◯ D ◯
4. A ◯ B ◯ C ◯ D ◯

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}}\,dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}}\,dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

© Board of Studies NSW 2013

6) a) π rads = 180°

$\frac{\pi}{5}$ rads = $\frac{180}{5}$

$= 36^\circ$

b) $6\cos(3x-1)$

c) $-\sin(e^x)\times e^x$

d) $f'(x) = \frac{x\times\cos x - \sin x\times 1}{x^2}$

e) $y' = 2\sin x\cos x$

@ $x=\frac{\pi}{2}$ $m = 2\sin\frac{\pi}{3}\cos\frac{\pi}{3}$

$= 2\times\frac{\sqrt{3}}{2}\times\frac{1}{2}$

$= \frac{\sqrt{3}}{2}$

$y - \frac{3}{4} = \frac{\sqrt{3}}{2}\left(x-\frac{\pi}{3}\right)$

$y = \frac{\sqrt{3}}{2}x - \frac{\pi\sqrt{3}}{6} + \frac{3}{4}$

f) $\int\cos 3x\,dx = \frac{1}{3}\sin 3x + C$

g) $\int_0^\pi \sin\frac{x}{2}\,dx = \left[-\frac{1}{2}\cos\frac{x}{2}\right]_0^\pi$ ~~$= -\frac{1}{2}\cos\frac{\pi}{2} - -\frac{1}{2}\cos 0 = -\frac{1}{2}$~~ $\left[-2\sin\frac{x}{2}\right]_0^\pi$

h) $V = \pi\int y^2\,dx$

$= \pi\int_0^{\pi/3}\sec^2 x\,dx$

$= \pi\left[\tan x\right]_0^{\pi/3}$

$= \pi\left(\tan\frac{\pi}{3} - \tan 0\right)$

$= \sqrt{3}\,\pi$

Question 7 Answers — Yr 12 Ass Task 3 2014

a) Due to misprint ALL students received 2 marks

However.

$\int\sin(2x)\,dx = 2$ at $x=\frac{\pi}{2}$

$2 = -\frac{1}{2}\cos(2x) + c$

$2 = -\frac{1}{2}\cos(2\times\frac{\pi}{2}) + c$ (2)

$2 = -\frac{1}{2}\times -1 + c$

$2 = \frac{1}{2} + c$

$c = \frac{3}{2}$ $\therefore f(x) = -\frac{1}{2}\cos(2x) + \frac{3}{2}$

b) for $f(x) = 2\sin x$

i) $f(a) = 2\sin a$ $f(-a) = 2\sin(-a)$

$= -2\sin(a)$

$= -f(a)$

$\therefore f(a) = -f(-a)$

$\therefore$ odd function (1)

ii) for $-\pi < x < \pi$

as its ODD values NEGATE

$\therefore \int_{-\pi}^{\pi} 2\sin x\,dx = 0$ (1)

or $= \left[-2\cos x\right]_{-\pi}^{\pi}$

$= -2\cos\pi - -2\cos(-\pi)$

$= +2 - 2$

$= 0$

c) 

via sohcahtoa

$\cos\theta = \frac{4}{a}$ $\sin\theta = \frac{32}{b}$

i) $a = \frac{4}{\cos\theta}$ $b = \frac{32}{\sin\theta}$ (1)

$a = 4\sec\theta$ $b = 32\operatorname{cosec}\theta$

$\therefore L = a+b = 4\sec\theta + 32\operatorname{cosec}\theta$

ii) $L = 4(\cos\theta)^{-1} + 32(\sin\theta)^{-1}$

$\frac{dL}{d\theta} = -4(\cos\theta)^{-2}\times -\sin\theta - 32(\sin\theta)^{-2}\times\cos\theta$

$= \frac{4\sin\theta}{\cos^2\theta} - \frac{32\cos\theta}{\sin^2\theta}$ (3)

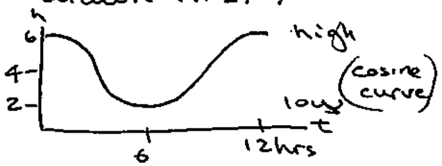
$= 4\tan\theta\times\sec\theta - 32\cot\theta\operatorname{cosec}\theta$

or via Quotient Rule.

(harder but same response)

d) $h = A\cos Bt + C$

two methods: both would be easier if diagram was drawn first.



$\therefore$ A is amplitude $= 2$ (1)

B is period $\frac{2\pi}{12} = \frac{\pi}{6}$ (1)

C is lift $= 4$ (1)

or algebraically

for B period is $\frac{2\pi}{n} = \frac{2\pi}{12}$

$= \frac{\pi}{6}$

when $t=0$ $h=6$

$6 = A\cos\frac{\pi}{6}\times 0 + C$

$A\times 1 + C$

$6 = A + C$ [1] (1)

when $t=6$ $h=2$

$2 = A\cos\frac{\pi}{6}\times 6 + C$

$2 = A\times -1 + C$

$2 = -A + C$ [2]

[1] + [2]

$8 = 2C$

$C = 4$ (1)

resub

$2 = -A + 4$

$A = 2$ (1)

$\therefore h = 2\cos\frac{\pi}{6}t + 4$

35 (a) $\frac{d}{dx}\left[\ln(x^2+3)\right] = \frac{2x}{x^2+3}$ ← (1)

(b) $y = x\ln x$

$\frac{dy}{dx} = \ln x + x \times \frac{1}{x}$

$= \ln x + 1$ ← (1)

(c) $\int_{\sqrt{5}}^{3} \frac{x}{x^2-4}\,dx = \frac{1}{2}\int_{\sqrt{5}}^{3} \frac{2x}{x^2-4}\,dx.$

$= \frac{1}{2}\left[\ln(x^2-4)\right]_{\sqrt{5}}^{3}$ ← (1)

$= \frac{1}{2}\left[\ln 5 - \ln 1\right]$

$= \frac{1}{2}\ln 5$ ← (1)

(d) $\text{Area} = \int_0^a \frac{1}{x+1}\,dx$

$3 = \left[\ln(x+1)\right]_1^a$ ← (1)

$3 = \ln(a+1) - \ln 1$

$3 = \log_e(a+1)$ ← ($\frac{1}{2}$)

$a+1 = e^3$

$a = e^3 - 1$ ← ($\frac{1}{2}$)

Multi Choice

1) D
2) C
3) D
4) A

e i) $f(x) = \frac{x}{\ln x}$

$f'(x) = \frac{\ln x \times 1 - x \times \frac{1}{x}}{(\ln x)^2}$

$= \frac{\ln x - 1}{(\ln x)^2}$

ii) Stat pts when $f'(x) = 0$

$\frac{\ln x - 1}{(\ln x)^2} = 0$

$\ln x = 1$

$x = e$

@ $x = e$ $\quad y = \frac{e}{\ln e}$

$= e$

x	1.5	e	2
$f'(x)$	-ve	0	+ve

∴ (e, e) is a min TP.

iii) $f(x)$ undefined when $\ln x = 0$

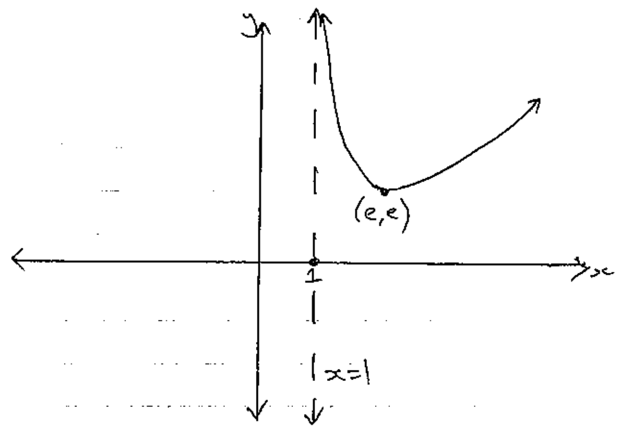
$x = 1$

as x approaches 1 from above

$f(x) \longrightarrow \infty$ eg. $f(1.001) =$

∴ $x=1$ is an asymptote.

iv)



f) $f(x) = \ln\left(\frac{x+2}{x}\right)$

$= \ln(x+2) - \ln x$

$f'(x) = \frac{1}{x+2} - \frac{1}{x}$

$= \frac{x - (x+2)}{x(x+2)}$

$= -\frac{2}{x(x+2)}$